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Magnetic Properties as Proxies for Geochemical Prediction in Mining Tailings: A Semi-Supervised Spatial Approach

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Abstract: Mine tailings require careful monitoring and management, but traditional geochemical characterization methods are costly and time-consuming. This study demonstrates that magnetic properties can serve as effective proxies for predicting copper concentrations in mine tailings through an innovative spatial modeling approach. Analysis of magnetic and geochemical measurements from a Chilean copper mine tailing showed that magnetic properties combined with spatial modeling techniques could predict copper concentrations with high accuracy ($R^2 = 0.873 \pm 0.085$). The spatial distribution of magnetic properties revealed coherent patterns that effectively predicted geochemical characteristics. This approach substantially reduces characterization costs compared to traditional methods while maintaining accuracy. Our findings establish magnetic properties as valuable screening tools for tailings characterization, offering mining operations a cost-effective approach to environmental monitoring and management.

Keywords: tailings geochemistry; magnetic susceptibility; spatial modeling; machine learning; environmental monitoring; mining waste management

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1. Introduction

Mining activities, particularly in copper extraction, generate substantial volumes of tailings that pose significant environmental challenges [1–3]. In general, mine tailings represent one of the largest waste streams globally, with production exceeding 14 billion tonnes annually [3]. Copper mining operations generate over half of this volume, creating extensive environmental liabilities that cost the mining industry an estimated USD 20 billion yearly in management and remediation efforts [2]. The scale and complexity of these waste facilities demand efficient characterization methods to support risk assessment and mitigation strategies [1,4]. However, comprehensive monitoring remains challenging due to the size of tailings deposits and the costs associated with traditional analytical approaches [5,6].

Tailings often contain elevated concentrations of heavy metals and other potentially harmful elements that require careful monitoring and management [4,7,8]. Traditional characterization methods are highly dependent on extensive geochemical analyses, which are both time-consuming and costly, especially when comprehensive spatial coverage is needed for large tailings facilities [5,9].

Magnetic properties have emerged as promising proxies for rapid environmental assessment, offering potential cost-effective alternatives to conventional geochemical analyses. Previous studies have demonstrated strong correlations between magnetic parameters and heavy metal concentrations in various environmental contexts. References [2,10,11] reported correlations between magnetic susceptibility and heavy metals in soils with R^2 values of 0.65–0.75, establishing the fundamental relationship between magnetic properties and metal content. Further studies by [11] demonstrated similar correlations in urban environments and highlighted the influence of anthropogenic activities on magnetic signatures. In mining contexts [12], found that magnetic measurements could effectively trace industrial contamination from Fe-Pb mining operations in alluvial soils, while [13] validated these techniques for characterizing mine tailings. In general, the application of magnetic techniques for tailings characterization has several advantages in terms of their measurement speed, cost-effectiveness, and non-destructive nature [12,13].

The relationship between magnetic properties and metal concentrations stems from shared mineralogical origins and weathering processes within ore deposits. Magnetic susceptibility correlates with heavy metal content because ferromagnetic minerals frequently co-occur with metal sulfides in primary mineralization. During weathering and oxidation, these minerals undergo parallel transformation processes that preserve these relationships. Seminal studies by Dold [8] demonstrated how magnetic properties track mineralogical changes during sulfide oxidation, while Hanesch and Scholger [10] established quantitative correlations between magnetic susceptibility and metal concentrations in mining-impacted soils.

The complex spatial distribution of geochemical properties within tailings facilities presents challenges for environmental monitoring and remediation planning due to the difficulties of modeling their contents. Traditional sampling approaches often struggle to capture this spatial variability effectively, leading to potential uncertainties in risk assessment and management strategies [14,15].

In general, spatial variability in tailings requires sampling frequencies that satisfy the Nyquist–Shannon theorem to capture heterogeneous distributions [1]. Traditional discrete sampling often misses critical spatial patterns due to density limitations [14,15]. Higher-resolution magnetic measurements enable three-dimensional characterization of these spatial variations while remaining cost-effective [10].

The integration of magnetic measurements with advanced spatial modeling techniques offers a promising pathway to address these challenges, potentially enabling a more comprehensive characterization of tailings facilities while reducing the required number of expensive chemical analyses [2,6,16].

This study presents a novel application of a semi-supervised spatial approach that leverages magnetic properties as predictive tools for geochemical characteristics in copper mine tailings. Our methodology combines high-resolution magnetic measurements with selected geochemical analyses, employing machine learning techniques to establish predictive relationships while explicitly accounting for spatial dependencies [17,18]. By incorporating spatial features, our objective is to develop a practical framework for tailings characterization that balances accuracy with cost-effectiveness.

The primary objectives of this research are to: (1) evaluate the effectiveness of magnetic properties as proxies for geochemical parameters, specifically copper concentration, in copper mine tailings, (2) develop and validate a spatial prediction framework that optimizes the use of limited geochemical data, and (3) assess the reliability and limitations of magnetic-based predictions across different spatial scales and depths. This work addresses a critical gap in tailings characterization methodology [2], potentially offering mining operations a more efficient approach to environmental monitoring and management.

2. Materials and Methods

Before delving into the detailed methodology, we note that all computational aspects of this study were implemented in the Python programming language using scikit-learn [19] for machine learning operations, pandas [20] for data manipulation, and matplotlib [21] for visualization. Spatial operations relied on custom implementations of IDW and nearest neighbor calculations.

2.1. Data Description

The data set analyzed in this study was collected at a copper mine tailings facility in the Atacama Desert, Chile, located at latitude 24°9′58.33″ S and longitude 69°2′33.36″ W at an elevation of 3200 m above sea level. As described by Lam et al. [2], the study area comprises a tailings terrace measuring 160 m × 80 m, which was active from 1995 to 2006. The original sampling and analysis program was conducted as part of the CORFO-INNOVA project (08CM01-05), which focused on developing integrated magnetochemical technologies for heavy metal remediation in mining environmental liabilities. While these are not the only data collected in this project, we focus on this particular subset to develop our semi-supervised spatial model. In this context, we note that the selection of magnetic features used for the model was based on the availability of data from this project, and thus, we did not have direct control over the data-collection process.

The sampling design followed a regular grid pattern with 80 measurement points. At each point, measurements and samples were collected at three distinct depths: 0–10 cm (depth a), 10–20 cm (depth b), and 20–30 cm (depth c).

The sampling grid used a 7 m spacing in the direction of the tailings walls (N05E) and a 15 m spacing in the W-E direction (N85E), providing comprehensive coverage of the internal area while accounting for site constraints [2]. This non-random sampling design aimed to achieve representative sampling across both directions [2,14]. The sampling density of 15 m × 7 m was selected based on two key factors—the expected scale of geochemical variability in tailings deposits and operational considerations from previous studies at similar facilities.

Magnetic property measurements included field and laboratory analyses. Magnetic susceptibility field measurements were performed using an SM-30 portable susceptometer with a precision of 1×10^{-7} SI. Laboratory analyses, performed on selected samples, included mass-specific magnetic susceptibility (χ), anhysteretic remanent magnetization (ARM), hysteresis parameters (Hc, Hcr), saturation magnetization (Js), remanent magnetization (Jr), saturation isothermal remanent magnetization (SIRM), and magnetic susceptibility at different frequencies (F1, F3).

Due to economic constraints, geochemical analyses were performed on a subset of 33 samples of several elements [2]. In particular, our analysis focused specifically on predicting copper concentrations, which ranged from 8181 to 17,203 mg/kg at the sample locations.

2.2. Three-Dimensional Inverse Distance Weighting

To address the challenge of missing geochemical data, we implemented a three-dimensional Inverse Distance Weighting (IDW) interpolation method [22]. The IDW method was selected for its ability to maintain spatial continuity while accounting for the anisotropic nature of the sampling grid and the vertical stratification of the tailings.

For any unsampled location $\mathbf{x}_0 = (x_0, y_0, z_0)$, where x_0, y_0, and z_0 represent the spatial coordinates of the location, the interpolated value $\hat{Z}(\mathbf{x}_0)$ was calculated as:

$$\hat{Z}(\mathbf{x}_0) = \frac{\sum_{i=1}^{n} w_i(\mathbf{x}_0) Z(\mathbf{x}_i)}{\sum_{i=1}^{n} w_i(\mathbf{x}_0)} \tag{1}$$

where $Z(\mathbf{x}_i)$ represents the known value at location $\mathbf{x}_i$, and the weights $w_i(\mathbf{x}_0)$ are defined as:

$$w_i(\mathbf{x}_0) = \frac{1}{d(\mathbf{x}_0, \mathbf{x}_i)^p} \tag{2}$$

with $d(\mathbf{x}_0, \mathbf{x}_i)$ being the Euclidean distance between the points $\mathbf{x}_0$ and $\mathbf{x}_i$ in the three-dimensional space:

$$d(\mathbf{x}_0, \mathbf{x}_i) = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2 + (z_0 - z_i)^2} \tag{3}$$

The power parameter p was set to 2, following common practice in geospatial interpolation based on established geochemical dispersion patterns in porous media [23–25]. This value reflects the physical process of diffusive transport while avoiding over-smoothing of local variations [26]. The focus on copper concentration as the primary prediction target stems from its role as the main constituent of economic interest, its demonstrated correlation with magnetic properties in previous studies [1,2], and its importance as an environmental indicator for this type of facility [3,4]. Finally, to prevent numerical instabilities, we implemented a minimum distance threshold of 10^{-10} units [27].

The choice of IDW interpolation to complete our missing geochemical data warrants careful consideration within the broader context of semi-supervised learning approaches [28,29]. Although traditional label-spreading methods typically operate in feature space [30], our application required a method that explicitly accounts for spatial relationships in three dimensions. IDW interpolation effectively functions as a spatially aware label-spreading algorithm, where the influence of known copper concentrations diminishes with distance according to a physically meaningful relationship.

The power parameter in IDW provides a theoretically grounded way to control the locality of influence, analogous to the bandwidth parameter in traditional label-spreading algorithms [31]. In our application, setting this parameter to 2 reflects the inverse square relationship commonly observed in physical processes, including the dispersion of geochemical elements in porous media [23–25]. This choice balances local detail preservation with the need for smooth interpolation across unsampled regions.

2.3. Spatial Feature Engineering

Our predictive model incorporated spatial relationships through a feature engineering [32] approach that considered both the magnetic properties and their spatial distribution. The process began with a nearest neighbor analysis, where we identified the five nearest neighbors for each sampling point based on the Euclidean distance on the horizontal plane. We note that we always include the point itself in these calculations. Thus, we used a total of six points to calculate these values.

We note that including the point itself could reduce the sensitivity to nearby variation, but including the closest measurement (i.e., distance of zero) makes sense from an intuitive standpoint, as it would be expected to be highly related to the actual measurements of copper in the sample.

For each magnetic measurement, we calculated three types of spatial features. First, we computed the mean value of the K nearest neighbors to capture the general magnetic characteristics of the surrounding area (*K-NN mean*). Second, we determined the standard deviation between these neighbors to quantify local variability (*K-NN standard deviation*). Third, we calculated a distance-weighted mean using the inverse distance weighting, giving more importance to the closer neighbors in the sample space (*K-NN IDW*). We note that these values are computed using the standard formulas for the mean, standard deviation, and IDW restricted to the K closest points.

This spatial feature engineering effectively transformed each magnetic measurement into three additional features that captured different aspects of its spatial distribution. The approach allowed our model to consider not just the magnetic properties at each point but also how these properties varied in the surrounding space, potentially revealing underlying spatial patterns in the geochemical distributions we aimed to predict.

2.4. Predictive Model

Our predictive approach employed a random forest regression model to establish relationships between magnetic properties and copper concentration. The feature set combined the original magnetic measurements with their engineered spatial features (K-NN mean, K-NN standard deviation, and K-NN IDW). In total, our model had 57 features, including the basic features and their spatial versions.

We configured the random forest model [33] with 100 trees using scikit-learn's implementation [19], maintaining default parameters for tree depth and split criteria to prevent overfitting. Prior to model training, we applied a standard scaling transformation to all input features [34], ensuring that magnetic measurements of different magnitudes could contribute equally to the prediction process. We performed this standardization process to facilitate comparison with other machine learning models.

The random forest algorithm proved particularly suitable for this application due to several key characteristics. First, it inherently handles the non-linear relationships [35] that often exist between magnetic properties and geochemical concentrations. Second, its ensemble nature provides robust predictions even with the relatively high-dimensional feature space [36] created by our spatial feature engineering approach. Third, it naturally manages the potential correlations between original measurements and their spatial derivatives through its tree-based structure [37]. Furthermore, the algorithm's feature importance metrics offer valuable insights into which magnetic properties and spatial features most strongly influence geochemical predictions [38].

We also note that we did not perform any dimensionality reduction nor did we apply any explicit regularization technique, as random forest models work as a form of regularization [39]. Finally, while more alternative algorithms like Gradient Boosting [40] and Support Vector Regression [41] exist, random forest provided good performance with minimal tuning required, making further optimization unnecessary for the purposes of our work, which is showing that magnetic properties work as proxies for copper concentration.

2.5. K-Fold Cross-Validation

Our modeling approach combined the aforementioned random forest regression method with a cross-validation strategy designed to assess both predictive accuracy and spatial generalizability [42]. The cross-validation procedure employed a k-fold approach with $k = 10$, a standard choice to balance the bias–variance trade-off [43]. The cross-validation procedure was stratified by depth to ensure representative sampling across the vertical profile of the tailings facility. For each fold, we performed complete feature standardization using only training data to prevent data leakage [44], with the scaling parameters subsequently applied to the corresponding test set.

2.6. Comparison Baselines

To validate the performance of our proposed spatial modeling approach, we implemented several baseline models for comparative evaluation. These baseline models were selected to represent different regression techniques and provide a comparison point to assess the effectiveness of our methodology.

The baseline models included:

- **Random forest without spatial features**: This model utilized the original magnetic measurements without the spatial feature engineering approach. By removing the spatially derived features, we could directly assess the impact of our spatial feature engineering technique on predictive performance.
- **Linear regression with spatial features**: A standard linear regression model [45] was implemented using the same set of features, including the spatially engineered attributes. This model allowed us to evaluate the performance of a linear approach against the non-linear random forest method.
- **Regularized linear regression variants**: We included three regularized regression techniques to explore different approaches to feature selection and model complexity:
  - Ridge regression: Employs L2 regularization to prevent overfitting by penalizing large coefficient values [46].
  - LASSO regression: Uses L1 regularization for feature selection by driving some coefficients to zero [47].
  - Elastic Net regression: Combines L1 and L2 regularization, balancing feature selection and coefficient shrinkage [48].

We performed hyperparameter optimization for the regularized regression models (Ridge, LASSO, and Elastic Net), with a grid search-based approach to find optimal values. However, given the consistently low predictive performance of these linear models, as demonstrated in Table 1, a more exhaustive hyperparameter tuning would have been computationally inefficient and unlikely to yield substantial improvements [49].

Table 1. Performance metrics for the proposed model against several other baselines with 10-fold cross validation.

Element	R^2 (Mean ± Std)	RMSE (Mean ± Std)
Proposed model	0.873 ± 0.085	510.250 ± 116.564
Random forest without spatial features	−0.120 ± 0.105	1643.423 ± 324.325
Linear regression with spatial features	−0.119 ± 0.410	1590.497 ± 259.956
Ridge regression with spatial features	0.011 ± 0.090	1544.203 ± 300.108
LASSO with spatial features	0.007 ± 0.139	1546.258 ± 310.730
Elastic Net with spatial features	0.014 ± 0.090	1542.689 ± 301.560

In particular, for Ridge, we tested α values from 10^{-3} to 10^3 over 20 points, with the best performance (α = 233.57) achieving R^2 = 0.011. LASSO optimization in the same range α yielded the best performance (α = 54.56) with R^2 = 0.007. Elastic Net was tested with α values from 10^{-3} to 10^3 and *l1_ratio* from 0.1 to 0.9, achieving R^2 = 0.014 with α = 10.0 and *l1_ratio* = 0.9. These results demonstrate that even with hyperparameter optimization, the linear models were unable to effectively capture the relationships in the data. The significant performance gap between these optimized linear models and our spatial random forest approach (R^2 = 0.873) validates our modeling choices and confirms the inherent non-linearity of the magnetic–geochemical relationships.

Each baseline model was configured with identical preprocessing steps, including standard scaling of features and the same 10-fold cross-validation strategy employed in our primary model. This consistent methodology ensured a fair comparison of the different modeling approaches. The implementation of these baseline models served two primary purposes: (1) to demonstrate the superiority of our spatial feature engineering approach, and (2) to provide insight into the performance characteristics of different regression techniques when applied to magnetic property-based geochemical prediction.

3. Results

3.1. Model Performance Overview

The proposed model achieved a mean R^2 of 0.873 ± 0.090 and RMSE of 533.036 ± 124.690 mg/kg in the cross-validation experiments, indicating high predictive capability across different spatial subsets of the data.

We show the obtained predictions and a comparison with the interpolated values from the data set by depth level in Figure 1.

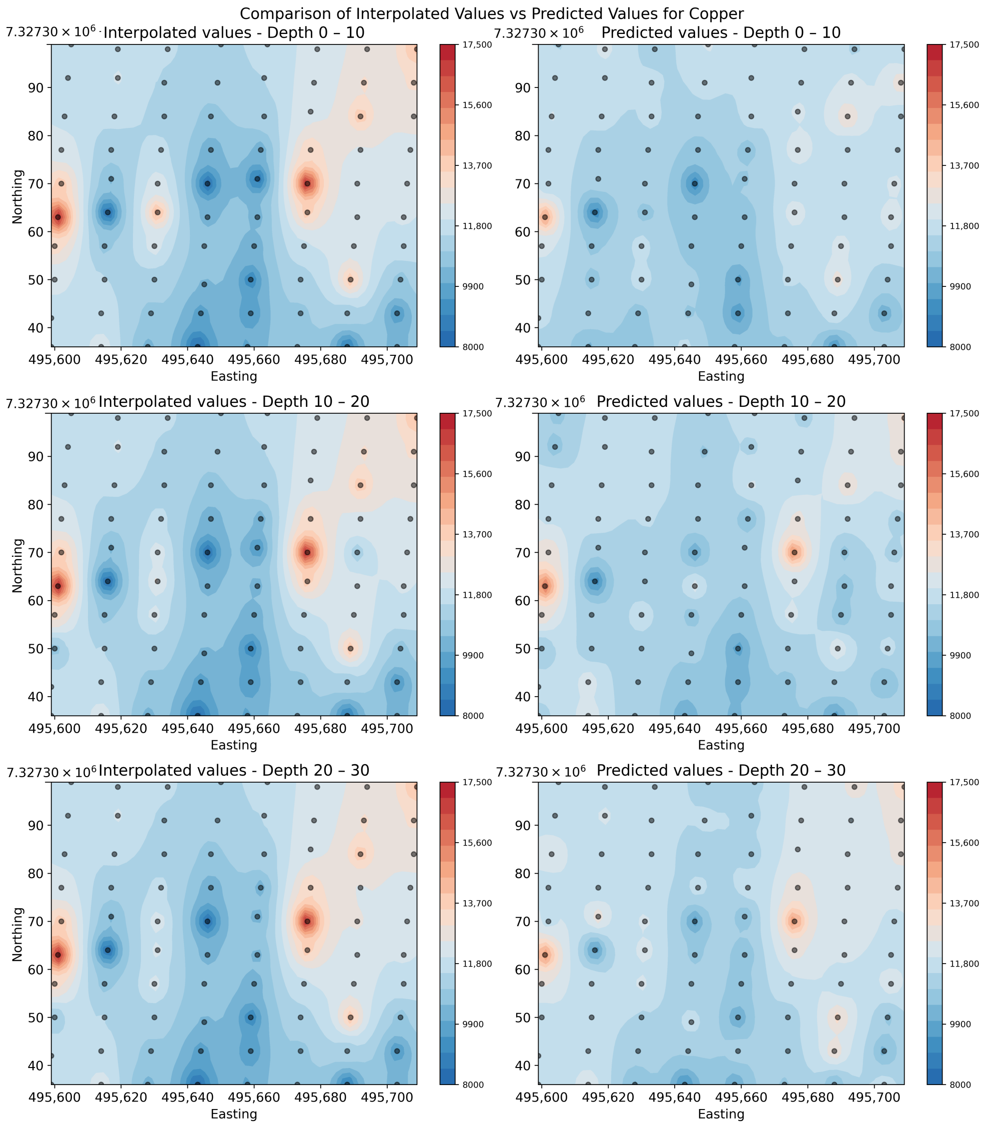

Figure 1. Interpolated values vs. predicted values by the proposed model.

We present an overview of the results for the linear regression models (basic linear regression, Ridge, LASSO, and Elastic Net) in Figure 2. Finally, we present a direct comparison between the prediction results of the random forest without spatial features and the proposed model in Figure 3. Note that this comparison is made with a 80/20 holdout validation for simplicity purposes.

The incorporation of spatial feature engineering represented a significant improvement over traditional approaches that rely solely on point measurements. By calculating three distinct types of spatial features—K-NN means, K-NN standard deviations, and K-NN IDW—our methodology captured multi-scale spatial relationships that significantly improved prediction accuracy.

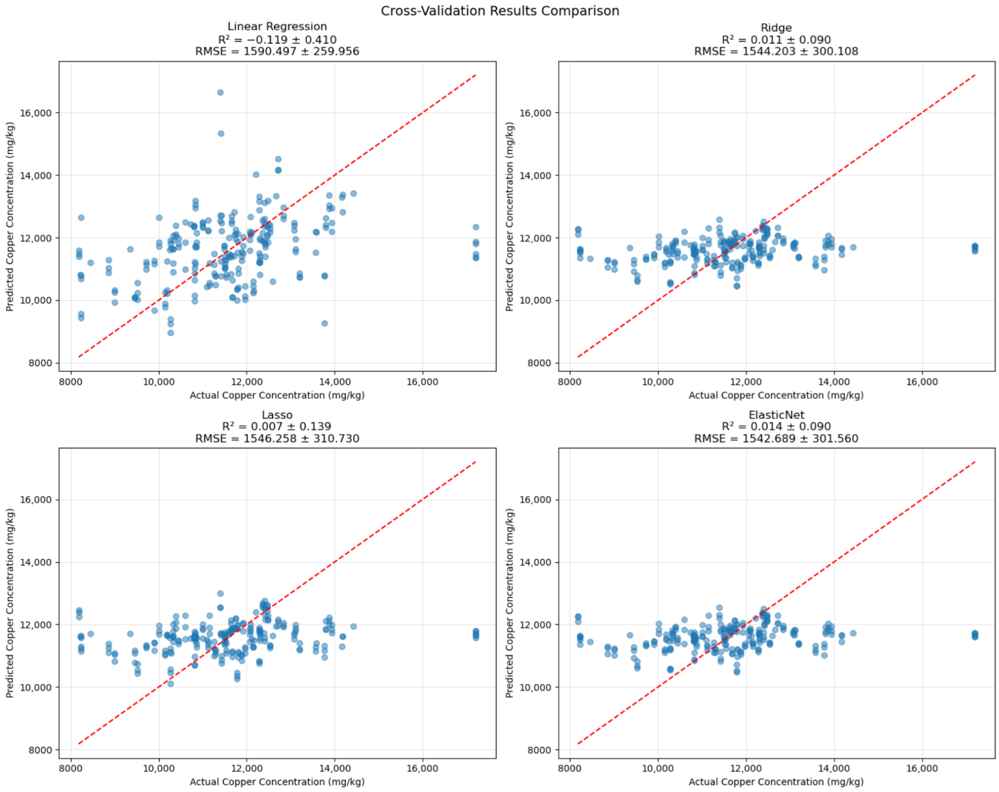

Figure 2. Results from linear regression with spatial features.

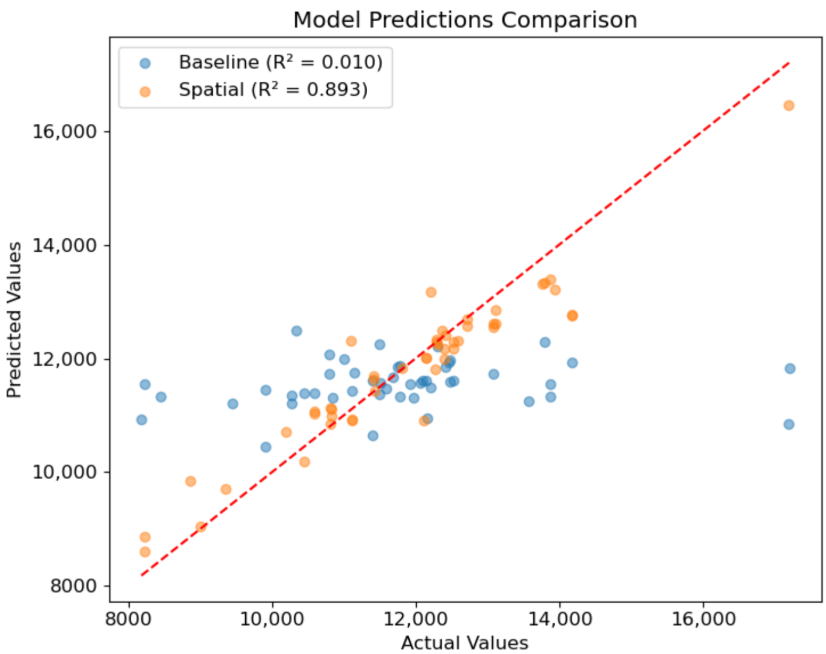

Figure 3. Comparison between a random forest without spatial features and the proposed model.

3.2. Feature Importance Analysis

The importance of individual features in tree-based regression models like the random forest used in this study is determined by evaluating the decrease in impurity (e.g., mean squared error) that each feature provides when used to split the data at internal nodes [35]. Features that result in larger reductions in impurity are assigned higher importance scores, as they are more influential in making accurate predictions.

The analysis of the importance of each feature revealed that Jr (μ Am^2/gr) mean neighbors was the most influential predictor, with a relative importance of 11.5%. The spatially derived features, particularly those based on magnetic susceptibility measurements, demonstrated strong predictive power. The top five most important features for copper prediction were:

- Jr (μAm^2/gr) (K-NN mean): 0.115 ± 0.024.
- Mass (g) (K-NN standard deviation): 0.063 ± 0.017.
- SIRM5Oe (nAm^2) (K-NN mean): 0.062 ± 0.022.
- Bsus F1 (microSI) (K-NN mean): 0.045 ± 0.014.
- Bsus F3 (microSI) (K-NN mean): 0.043 ± 0.012.

These results support the validity of using magnetic properties as proxies for geochemical prediction in mining tailings while highlighting the importance of considering spatial relationships in the modeling approach. We show the ranking of all features by importance in Figure 4.

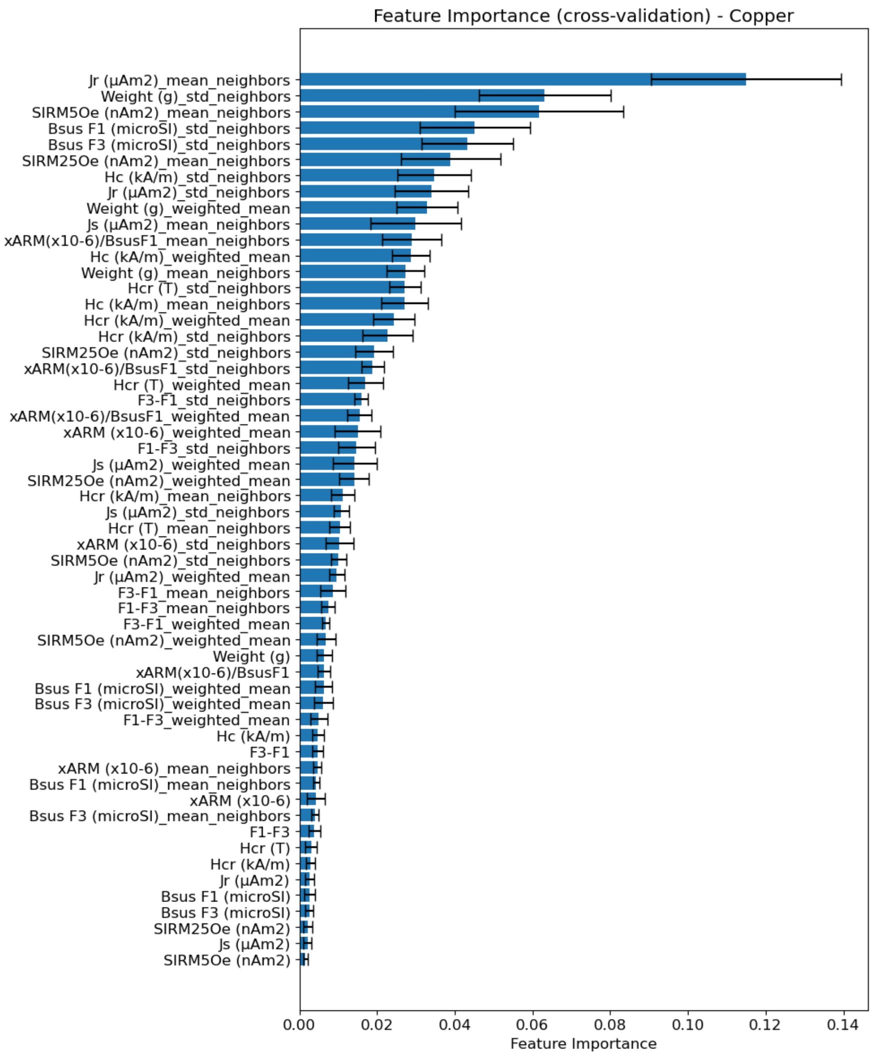

Figure 4. Feature importance ranking for the proposed model. The x-axis represents normalized feature importance (as a percentage) and the y-axis represents the different features used in the model.

The pie chart presented in Figure 5 shows the relative contribution of different feature types to the model's prediction of copper concentration.

The *K-NN mean* spatial features make up the largest proportion at 37.9% of the total feature importance. This indicates that the average values of magnetic properties in the surrounding neighborhood are highly predictive of copper concentrations.

The *K-NN standard deviation* spatial features contribute 37.0% to the overall feature importance. This suggests that the local variability in magnetic properties also plays a significant role in predicting copper levels, likely capturing spatial heterogeneity within the tailings.

The *K-NN IDW* spatial features account for 20.2% of the importance, demonstrating that the distance-weighted averages of magnetic properties provide additional predictive power beyond just the local means and standard deviations.

Finally, the original features, which are the raw magnetic measurements without any spatial processing, contribute the lowest proportion at 4.9%. This lower importance could be caused by the fact that we included the values of the point itself in the computations of the K-NN features. However, we note that when removing the spatial features altogether, the model does not perform as well as the full spatial model. Given the low importance of the original features, this highlights the value added by the spatial feature engineering approach used in the modeling.

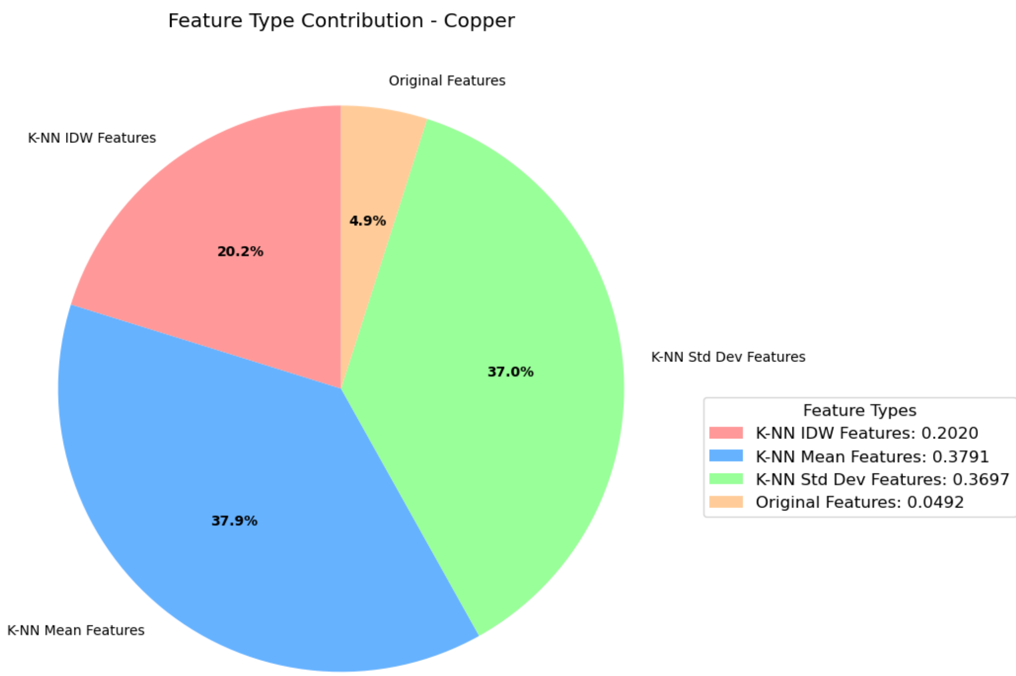

Figure 5. Pie chart describing the importance of different groups of features in the model.

4. Discussion

4.1. Interpretation of Magnetic–Geochemical Relationships

The strong correlation between magnetic susceptibility and copper concentration suggests that magnetic measurements can serve as effective proxies for metal content assessment. This relationship appears to be driven by the mineralogical association between magnetic minerals and copper-bearing phases in the original ore, as well as subsequent alterations within the tailings environment [1,8].

The relative importance of magnetic parameters in the prediction of geochemical concentrations provides insight into the underlying mechanisms. The dominance of Jr (μAm^2/gr) mean value across the nearest neighbors as a predictive feature suggests that the characteristics of remanent magnetization are particularly sensitive to the mineralogical changes associated with variations in metal concentration [50,51]. Furthermore, the K-NN mass standard deviation was also relevant, reflecting that variability in the sample mass, despite consistent sampling strategies, is related to the copper concentration in the samples.

Moreover, the high importance of SIRM could be caused by the common link between it and copper concentrations to the original sulfide mineralogy of porphyry copper deposits, as ferrimagnetic minerals such as magnetite commonly occur alongside copper sulfides during primary mineralization and oxidation processes [1]. Next, the significant contribution of features derived from magnetic susceptibility measurements at different frequencies (Bsus F1 and F3) indicates that grain size-dependent magnetic properties also play a crucial role in these relationships [52].

Furthermore, the strong performance of spatially engineered features in our predictive model underscores the importance of considering neighborhood effects in magnetic–geochemical relationships. This spatial dependence likely reflects both the original depositional patterns of the tailings and the subsequent development of geochemical gradients through processes such as oxidation, dissolution, and precipitation [1,8]. The effectiveness of these spatial features in prediction suggests that magnetic measurements can capture not only local metal concentrations but also broader patterns of geochemical evolution within the tailings facility.

4.2. Model Performance and Predictive Capabilities

Our spatial modeling approach demonstrated robust performance in predicting geochemical concentrations from magnetic properties. The random forest model achieved high cross-validation performance (R^2 = 0.873 ± 0.090). The incorporation of spatial feature engineering proved crucial in improving predictive accuracy. The traditional approach of using raw magnetic measurements alone yielded substantially lower cross-validation performance ($R^2 = -0.120 \pm 0.105$) compared to the spatially enhanced model. By incorporating neighbor-based features and distance-weighted averages, our model captured the inherent spatial structure of both magnetic and geochemical variables. The importance of these spatial features, particularly the Jr (μAm^2/gr) K-NN mean with an average importance value of 0.115, suggests that the local spatial context provides essential information for accurate prediction.

Compared with previous studies on magnetic proxies in environmental assessment, our results demonstrate several advances. Although earlier work by Hanesch and Scholger [10] reported R^2 values of 0.65–0.75 for heavy metal predictions using magnetic susceptibility alone, our spatially enhanced approach achieved higher accuracy (R^2 = 0.873). Other previous models include the work by Karimi et al. [11], which managed to explain 54% of the variance in the data, compared to our 87.5%. The improvement can be attributed to both our comprehensive spatial modeling framework and the specific characteristics of mining tailings, where the magnetic–geochemical relationships are potentially stronger because of the common mineralogical origin of the materials [1,2]. However, we note that in other types of soils (e.g., alluvial soils), magnetic properties have been found to explain even higher levels of variance (R^2 = 0.92) [12].

The random forest algorithm's ability to handle non-linear relationships [36] proved essential for accurate prediction. The complex interactions between magnetic properties and geochemical concentrations, influenced by factors such as mineral transformation, pH gradients, and water movement, create inherently non-linear patterns that simpler linear models failed to capture adequately. The model's feature importance rankings provide valuable insights into these relationships, highlighting the relative significance of different magnetic parameters in predicting geochemical concentrations.

Finally, we acknowledge that a systematic evaluation of different parameter ranges (e.g., K = 3 to 7 neighbors, or $p = 1$ to 3 for IDW) could provide additional insights. However, our focus was on developing a practical methodology, and the strong performance achieved with these straightforward parameter choices ($R^2 = 0.873 \pm 0.085$) suggested that complex parameter optimization was unnecessary.

4.3. Implications for Tailings Characterization

The implementation of magnetic properties as proxies for geochemical characterization offers significant practical and economic advantages for tailings management [2]. We note that for our study area of 10,000 m^2, the magnetic approach enabled comprehensive spatial coverage with 240 measurement points, whereas budget constraints would have limited a conventional geochemical survey to fewer than 33 sampling locations.

Current characterization methods like sequential extraction and spectroscopic analysis provide detailed compositional data but require extensive sample preparation, specialized equipment, and high analytical costs exceeding USD 150 per sample [5–7]. These constraints make comprehensive spatial characterization impractical for large facilities [2]. Alternative geophysical methods such as electrical resistivity tomography enable broad spatial coverage but lack direct correlation to chemical concentrations [53,54]. Magnetic susceptibility measurements uniquely bridge this gap by combining rapid, low-cost field acquisition with demonstrated relationships to geochemical parameters [55–57], enabling efficient screening and monitoring of large tailings facilities [58,59].

Moreover, the rapid measurement capabilities of magnetic surveys provide opportunities for more frequent monitoring of tailings facilities [55]. Field measurements of magnetic susceptibility require only minutes per location, allowing an experienced technician to collect hundreds of measurements per day. This efficiency enables the development of high-resolution temporal monitoring programs that would be prohibitively expensive using conventional geochemical methods [56]. The non-destructive nature of magnetic measurements [57] further supports repeated sampling at identical locations, facilitating the tracking of changes over time.

Integration of this approach into existing tailings management practices appears particularly promising for initial screening and ongoing monitoring applications [2,58]. The spatial patterns revealed by our analysis suggest that magnetic surveys can effectively identify zones of potential environmental concern, allowing for a more targeted application of detailed geochemical analyses. This targeted approach optimizes resource allocation while maintaining the ability to properly characterize the site.

The implications extend beyond operational cost savings to environmental risk management. The higher spatial resolution achievable through magnetic surveys enables better delineation of potential contamination pathways and more accurate assessment of environmental risks [59]. This improved spatial understanding can guide the design of more effective containment and remediation strategies [2,60]. The ability to quickly identify changes in tailings properties through repeated magnetic surveys also supports more proactive environmental management approaches.

For regulatory compliance, our findings suggest that magnetic surveys could complement, though not entirely replace, required geochemical monitoring programs. The demonstrated correlations between magnetic properties and metal concentrations provide a scientific basis for using magnetic measurements as screening tools to optimize the timing and location of regulatory compliance sampling [61]. This approach could lead to more efficient regulatory monitoring while maintaining or improving environmental protection standards. However, additional validation studies are needed before proper regulatory adoption.

4.4. Limitations

The current study, while demonstrating the potential of magnetic properties as proxies for geochemical characterization, has several limitations that should be addressed in future research. The sampling density of 15 m $\times$ 7 m, though sufficient for broad spatial characterization, may not capture fine-scale variability in tailings properties. Future studies should investigate the optimal sampling density through nested sampling designs that could better characterize spatial variability across multiple scales [14].

The temporal dimension represents another significant limitation of our current approach. The study provides a snapshot of tailings properties, but tailings facilities are dynamic systems [62,63] that evolve over time through oxidation, weathering, and various geochemical processes [64,65]. Long-term monitoring studies are needed to validate the stability of magnetic–geochemical relationships and assess their reliability for temporal change detection [53,54]. These studies should encompass seasonal monitoring to assess climatic variations, multi-year observations to track progressive oxidation effects, and event-based sampling to evaluate the influence of significant environmental changes [66–68].

There are other methodological limitations that warrant discussion. While magnetic surveys offer cost advantages, alternative field methods like portable XRF could provide complementary geochemical data. The SM-30 susceptometer's sensitivity (1×10^{-7} SI) proved adequate for tailings characterization, though more sensitive instruments might detect subtle variations in weakly magnetic phases.

Furthermore, cross-validation in spatially autocorrelated data presents challenges, as nearby points sharing similar properties violate independence assumptions. While our K-fold approach demonstrated model stability, true spatial generalizability requires validation across different facilities. Feature scaling, applied uniformly across models for comparison, had minimal impact on random forest performance but enabled fair evaluation of linear baselines.

The dominance of Jr mean neighbors in predicting copper likely reflects shared mineralogical origins in primary sulfides and parallel weathering transformations. Frequency-dependent susceptibility indicates magnetic grain size distributions that correlate with sulfide alteration patterns. Nearest neighbor averaging captures local trends in these transformation processes more effectively than point measurements alone, as chemical and physical heterogeneity typically occurs over 5–15 m scales.

We note that we selected IDW interpolation due to its simplicity and suitability for environmental data [24,69]. However, we acknowledge that IDW has important limitations as it tends to smooth out extreme values, potentially underestimating peak concentrations and assumes equal influence in all directions, which may not be true in tailings environments. While more complex methods like kriging [70] could provide additional statistical insights, IDW offered a good balance between accuracy and computational efficiency for our application of developing practical methods for tailings characterization.

Finally, the performance of our model must be considered within key limitations. Vertical variations in magnetic properties arise from multiple mechanisms beyond just geochemical changes—including physical compaction, mineral segregation during deposition [1], and variations in moisture content [52]. Although our results demonstrate strong correlations, establishing these as universal proxies would require validation across different tailings facilities and conditions [8,14,15].

4.5. Future Work

The current model focuses solely on copper concentrations. Thus, it would be useful to create individual models for the elements within the tailings environment or even a multi-output model [71,72] that handles several of these elements at the same time to leverage potential correlations between their geochemical distributions.

The application of our methodology to different types of tailings facilities requires further investigation. The current study focused on copper mine tailings in an arid environment, but the effectiveness of the approach can vary significantly with different ore mineralogy, processing methods, climatic conditions, weathering rates, and facility management practices. Understanding these variations will be crucial for broader application of the technique.

Model enhancement represents another important avenue for future work. Although our random forest approach proved effective, the integration of deep learning techniques [73] could potentially improve the precision of the prediction, particularly to capture complex spatial patterns [74,75]. The development of hybrid models that combine physical process understanding with machine learning could provide more robust and interpretable predictions [76].

5. Conclusions

This study demonstrates that magnetic properties can serve as effective proxies for predicting geochemical characteristics in copper mine tailings, offering a cost-effective approach to comprehensive site characterization.

First, our spatial modeling framework successfully leveraged magnetic measurements to predict copper concentrations with high accuracy (R^2 = 0.873 ± 0.090) while providing valuable insights into the three-dimensional distribution of contamination within the tailings facility. Second, our spatial modeling framework successfully integrates magnetic measurements with machine learning techniques, providing a novel approach to optimize sampling strategies and reduce characterization costs. Third, our feature importance analysis reveals that spatially derived magnetic features, particularly Jr ($\mu Am^2/gr$) mean neighbors, serve as the strongest predictors of copper concentration, offering new insights into magnetic–geochemical relationships in tailings environments.

Finally, we note that this study validates the use of magnetic properties as proxies for geochemical characteristics in mine tailings, offering a practical solution to the challenges of comprehensive site characterization. The demonstrated success of this approach in predicting copper concentrations, combined with its cost-effectiveness and efficiency, suggests significant potential for broader application in mining environmental management.

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